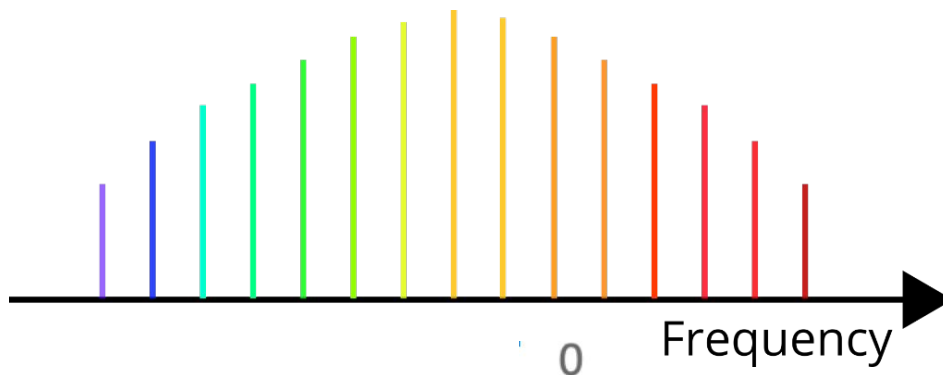




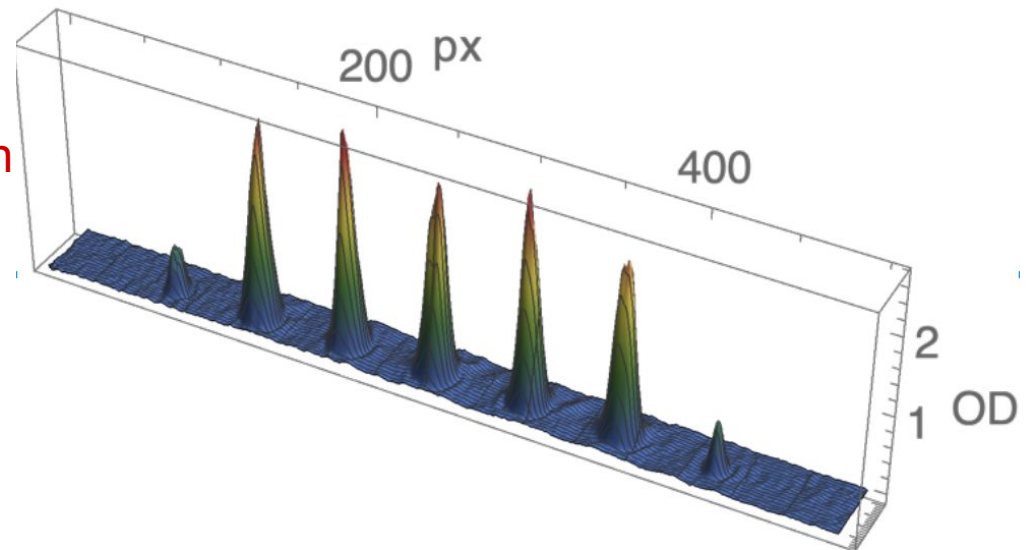
Bosonic “leg”: simulation of frequency comb with atomic comb of momentum components



Goal:

find/study non-classical correlations<sup>[OBJ]</sup> of atom numbers in momentum “teeth”

Contact interactions analogous to Kerr non-linearity (4-wave mixing)



To investigate the existence of quantum correlations in the emission of QCL-comb, we aim to engineer an atomic analog based on momentum states

T2.4: Arbitrarily filtered, multimode, Kapitza-Dirac interferometer (CNR; M1-M18)

T2.5: Generation and detection of quantum correlations (CNR; M1-M24)

T2.6: Controlling interactions to control quantum correlations (CNR; M18-M36)

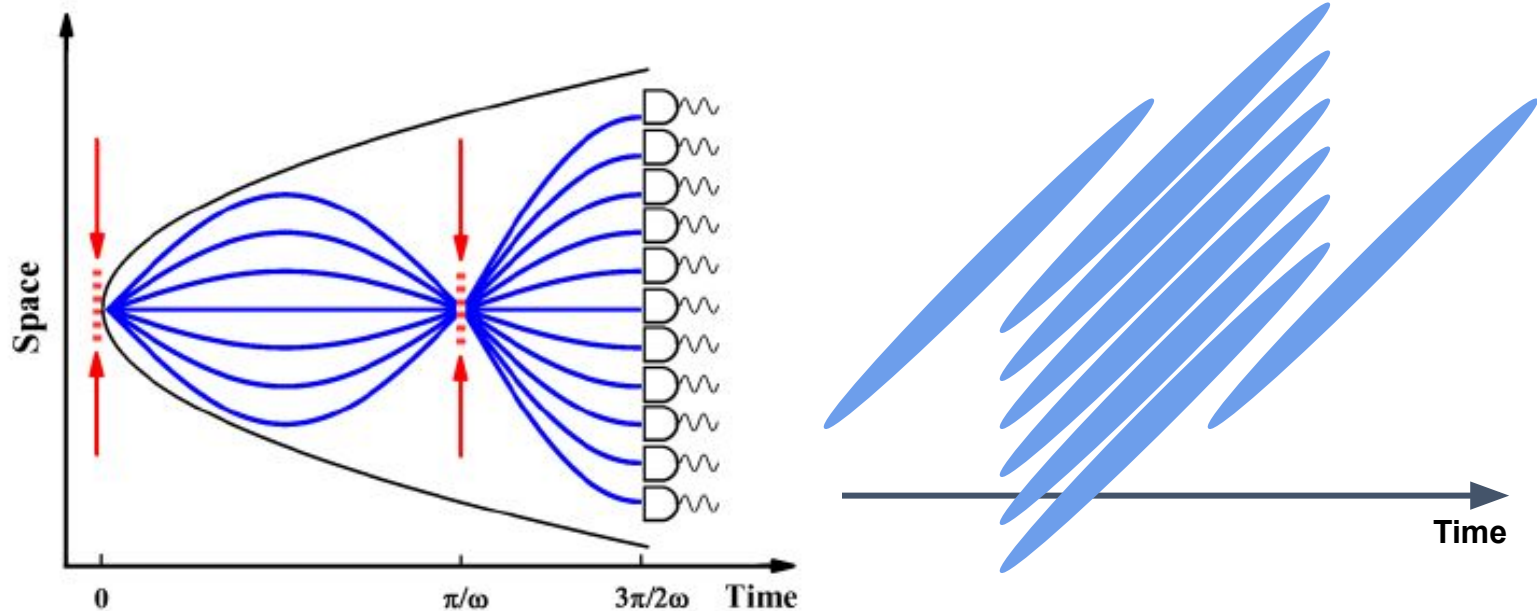
### T2.4: Arbitrarily filtered, multimode, Kapitza-Dirac interferometer (M1-M18)

The multiple momentum components corresponding to the comb photonic modes will be obtained from a single trapped Bose-Einstein condensate (BEC), by means of a Kapitza-Dirac pulse.

We will implement the established protocol of trapped Kapitza-Dirac interferometry using a BEC of  $^{87}\text{Rb}$  atoms. As a new resource, we will develop an arbitrary “momentum filter” to remove components at will. The use of light masks generated by DMD will grant arbitrary selections.

### D2.3: Implementation of Kapitza-Dirac multimode beam-splitter and interferometer (M18)

Proposed by Li et al. [PRL 113, 023003 (2014)],  
 partially realized in experiment by R.E. Sapiro et al. [PRA 79, 043630 (2009)]



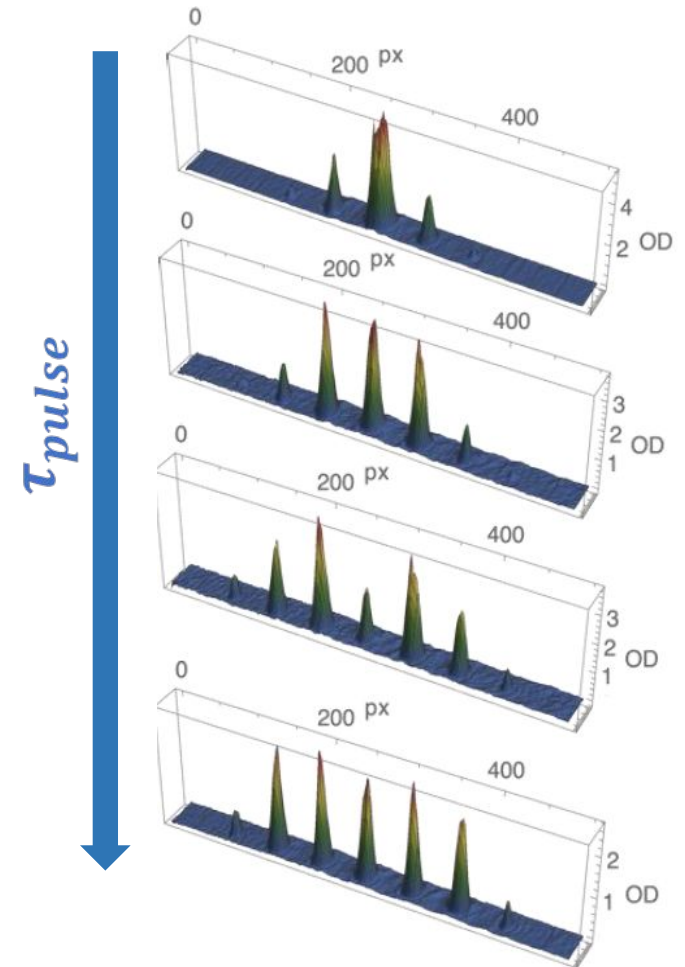
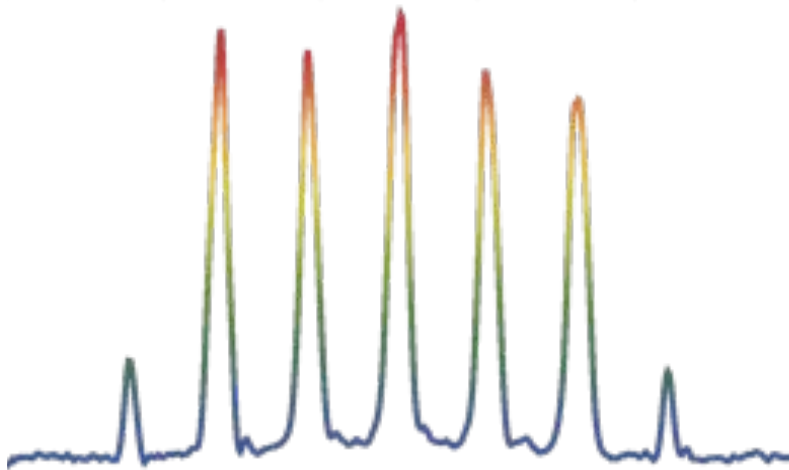
Our sample is elongated 3D condensate

Choice: split along short axis (stronger confinement) to minimize overlap time.

Kapitza-Dirac pulse

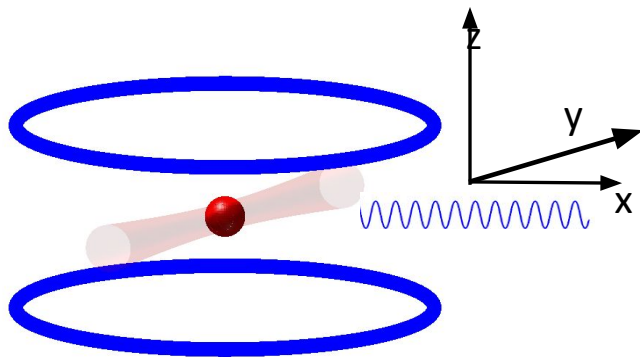


Comb of momentum states

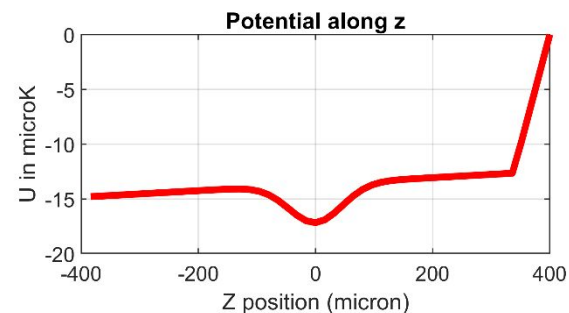
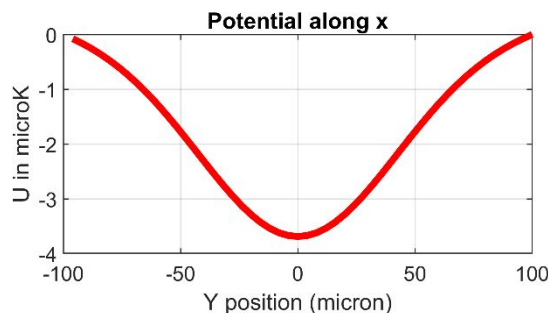
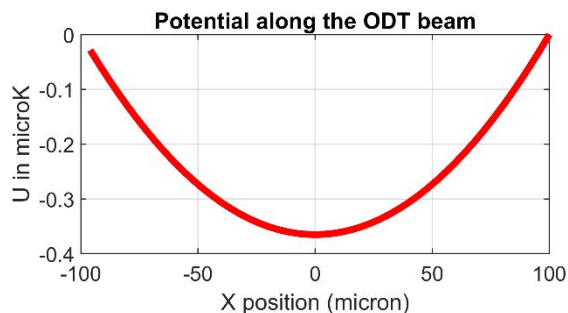


The comb of momentum states MUST be aligned along a normal axis of the harmonic oscillator trap

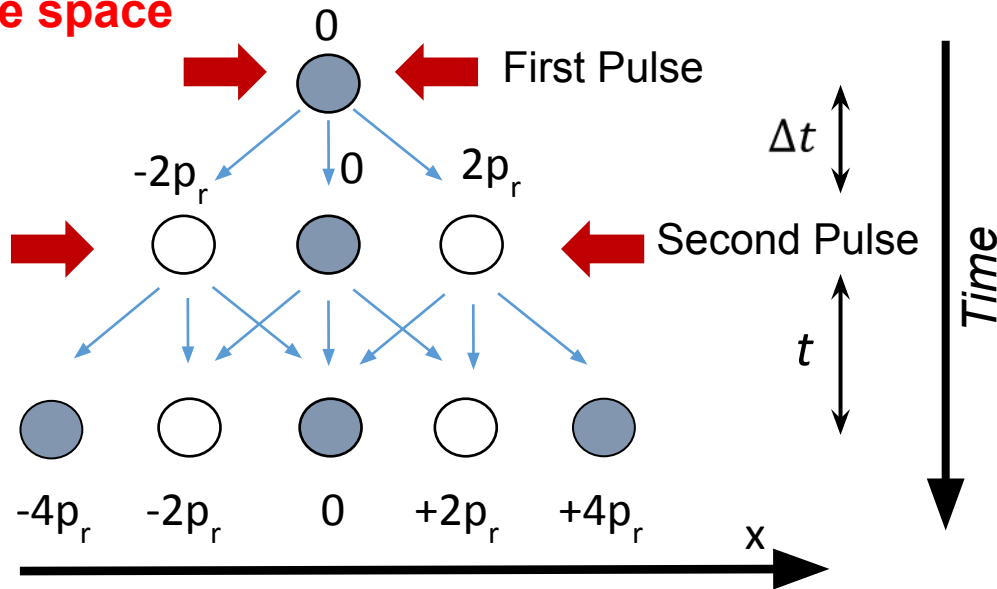
Magnetic quadrupole potential + single beam optical potential  
 Lattice direction (x) orthogonal to quadrupole axis (z) and optical trap wavevector (y)



Trap frequencies:  $(\omega_x, \omega_y, \omega_z) = 2\pi (70, 13, 65) \text{ Hz}$   
 $N = 3 \times 10^5$  atoms, Thomas-Fermi radii  $(R_x, R_y, R_z) = (6, 34, 7) \mu\text{m}$



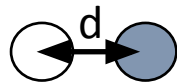
Free space



$$p_r = \hbar k$$

$$v_r = 3.8 \text{ mm/s}$$

Interference between two modes

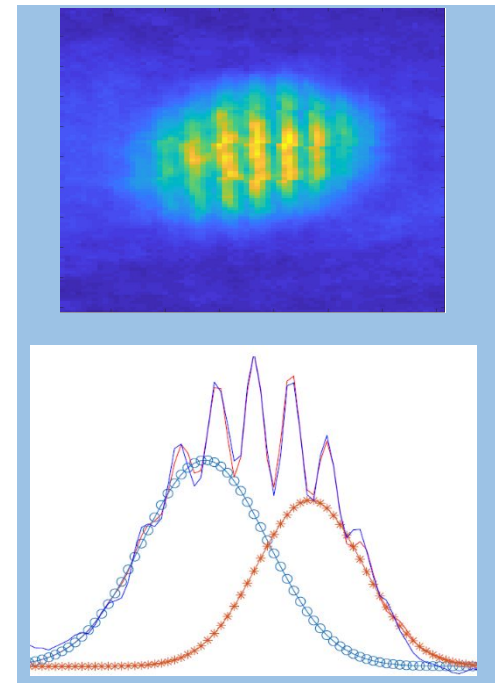


$$\varphi(r) = \varphi_l(r) + \varphi_r(r)e^{i\Phi}$$

$$n(r, t) = n_l(r, t) + n_r(r, t) + 2\sqrt{n_l(r, t)n_r(r, t)} \cos\left(\frac{md}{\hbar t}x + \Phi\right)$$

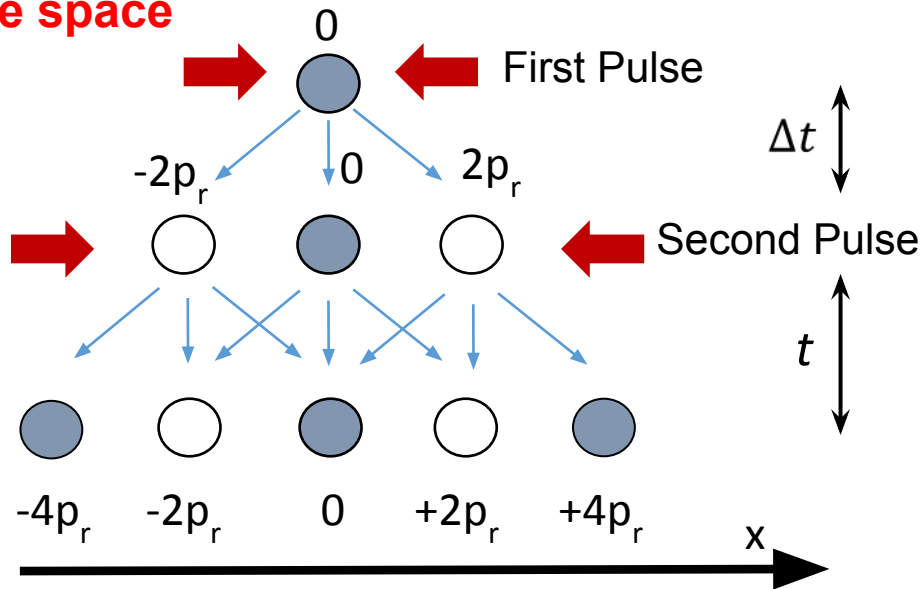
$$\lambda = \frac{\hbar t}{md}$$

$$d = 2v_r \Delta t$$

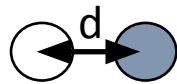




Free space



Interference between two modes



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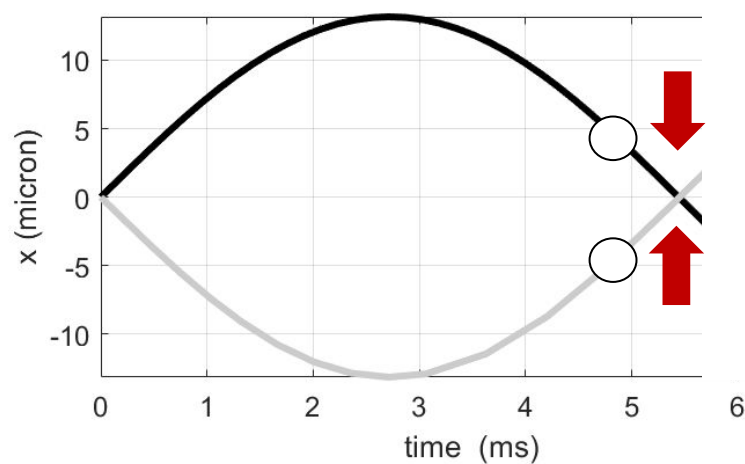
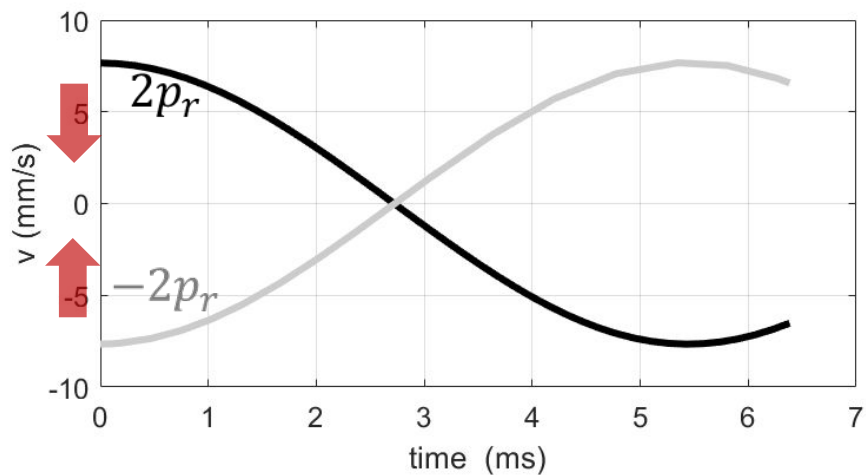
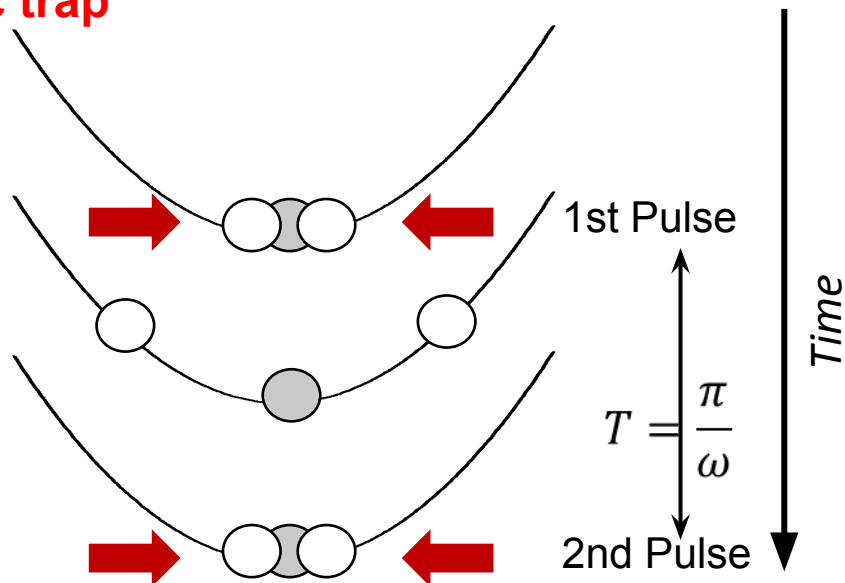
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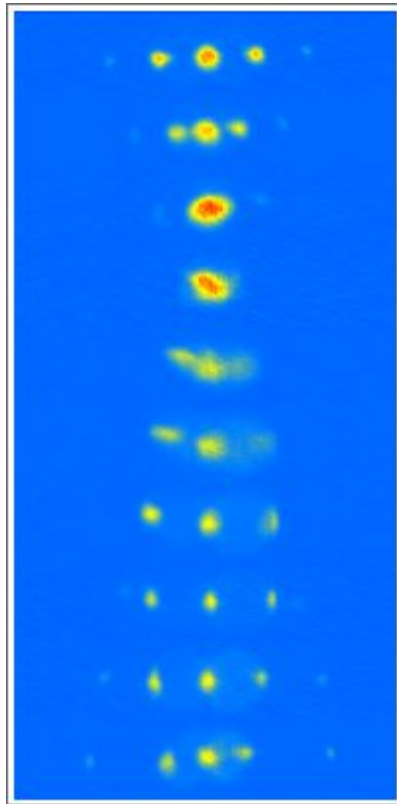
## Harmonic trap



Kapitza-Dirac pulse and subsequent evolution in trap

trap time [ms]

1  
2  
3  
..  
..  
..  
..  
8



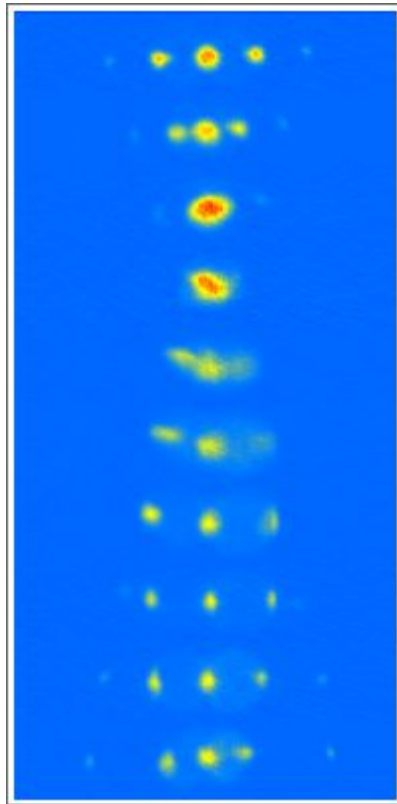
1st pulse,  $10\mu\text{s}$

Images after long TOF (32ms)  
→ momentum distribution

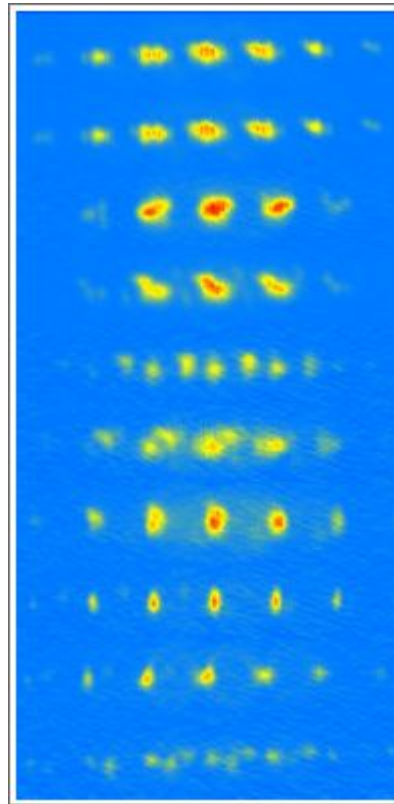
Kapitza-Dirac pulse and subsequent evolution in trap

trap time [ms]

1  
2  
3  
..  
..  
..  
8

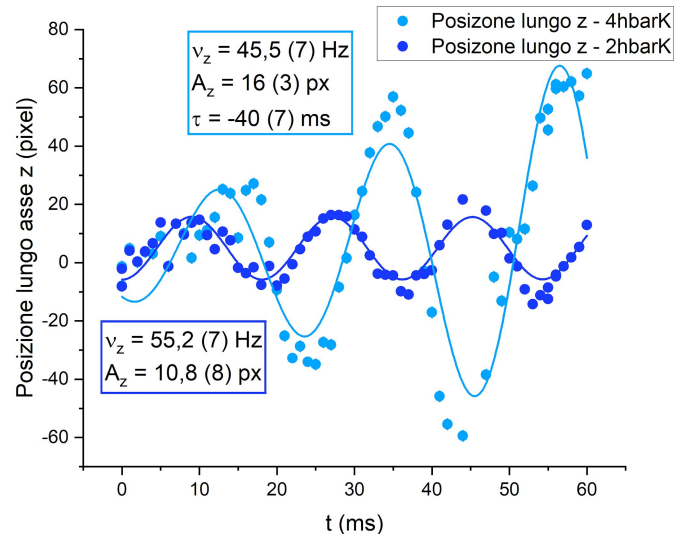
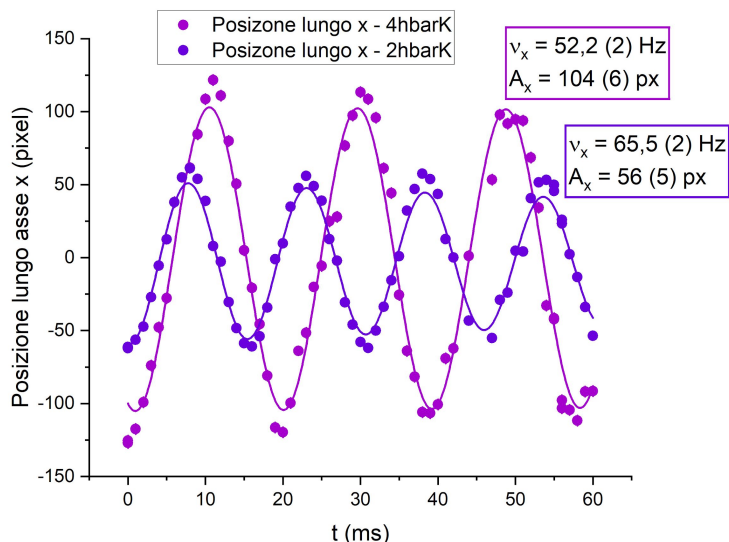


1st pulse,  $10\mu\text{s}$



2nd pulse,  $10\mu\text{s}$

Images after long TOF (32ms)  
→ momentum distribution



- Anharmonicity:  
since oscillation amplitude  $2v_R/\omega \sim$   
laser beam size,  
BEC at  $2v_R$ , frequency 52Hz  
BEC at  $4v_R$ , frequency 66Hz

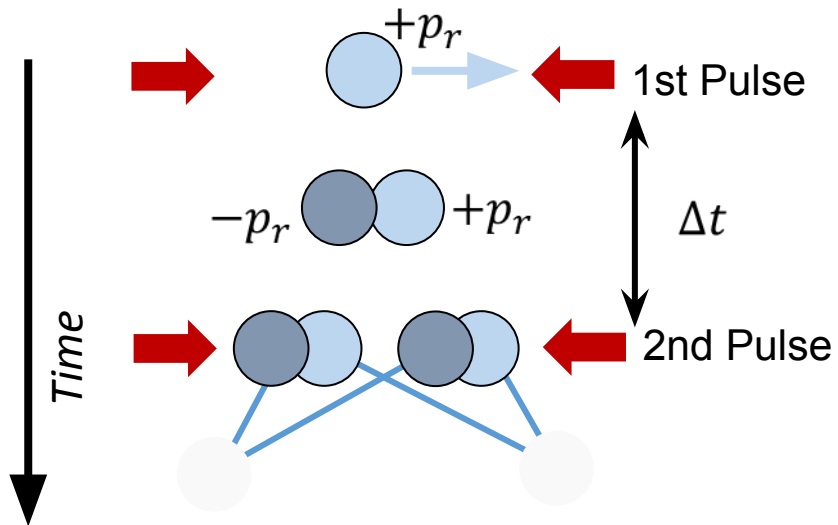
- Motion excited along transverse direction  
→ momentum components do not overlap after trap half-period

Solutions:

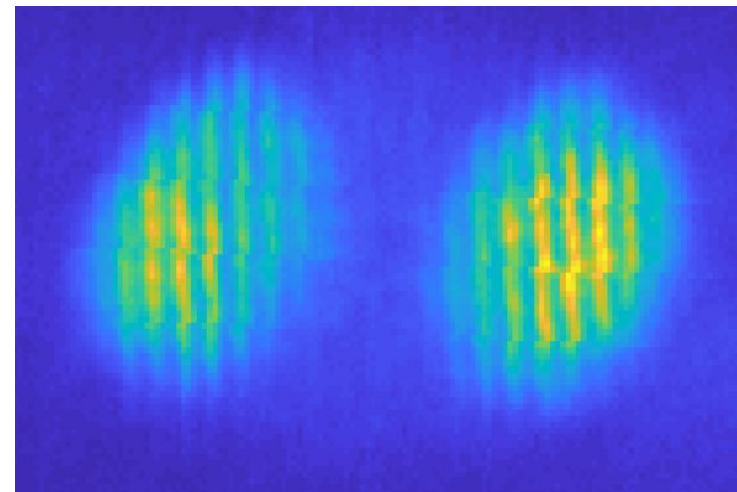
- reduce oscillation amplitude (easy)
- reduce lattice wavevector (more difficult)

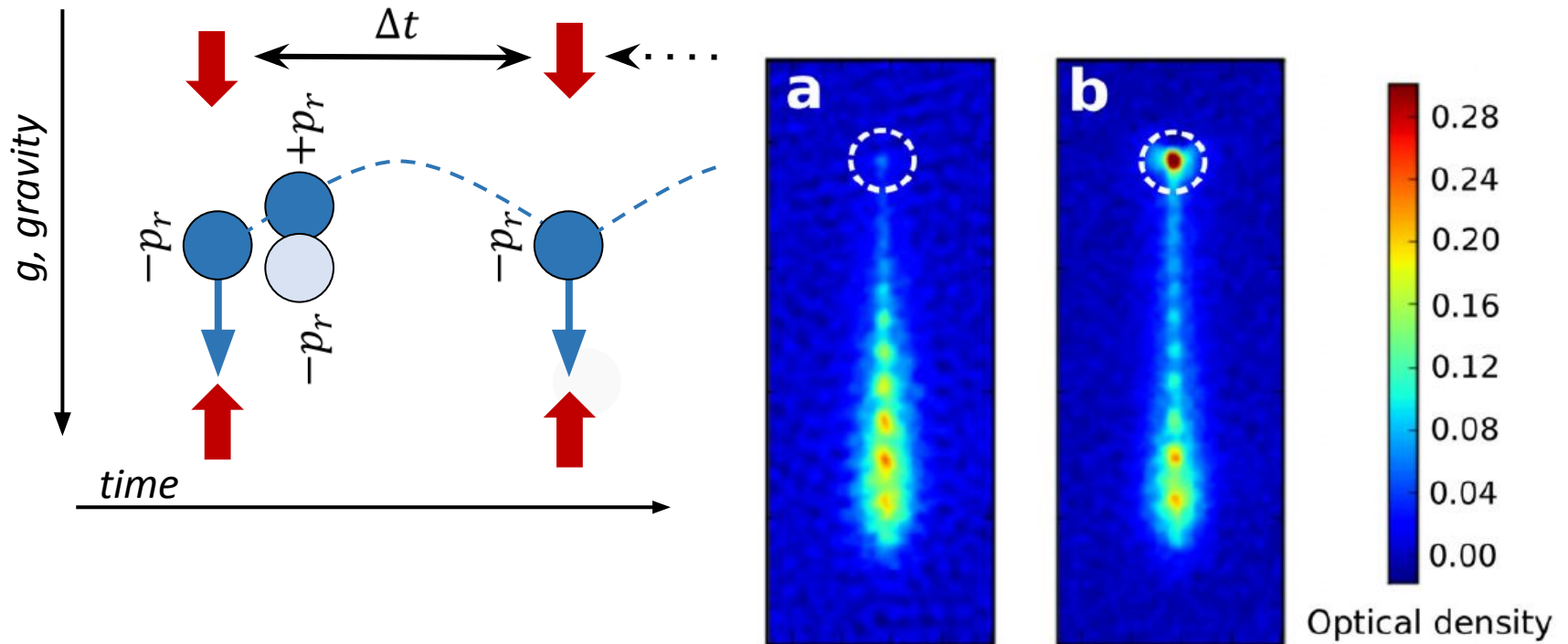


Lattice pulses along horizontal direction



Absorption imaging, after TOF

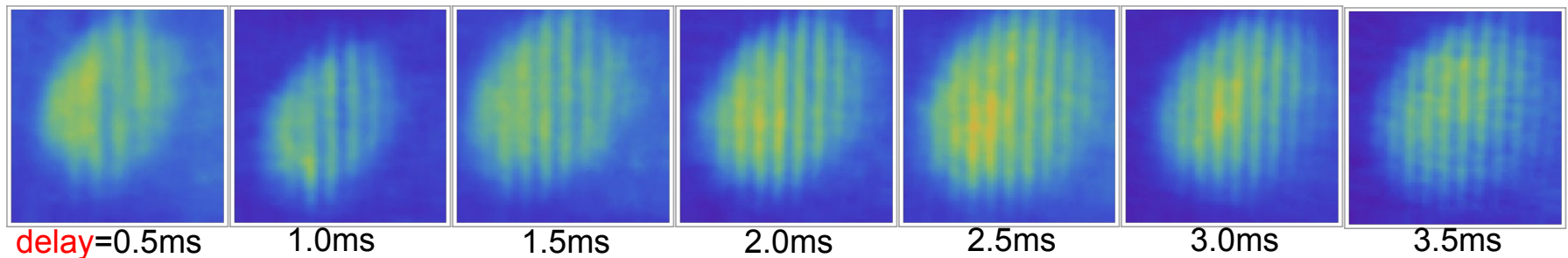
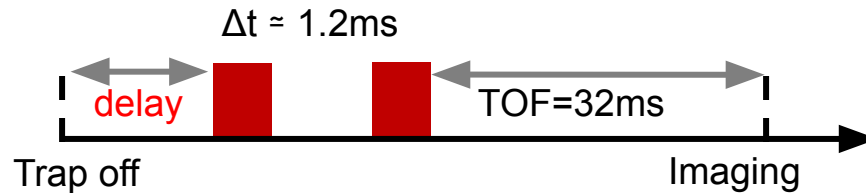




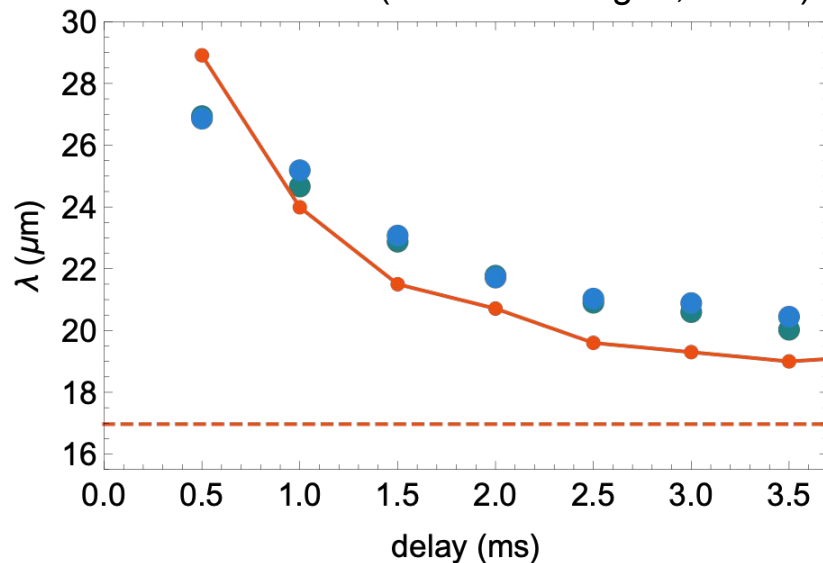
M. Robert-de-Saint-Vincent et al. EPL, 89 (2010) 10002

Multiple Bragg pulses: max efficiency at  $\Delta t = \text{fall time} \rightarrow \text{measure gravity (rel. unc. } 4 \cdot 10^{-4})$





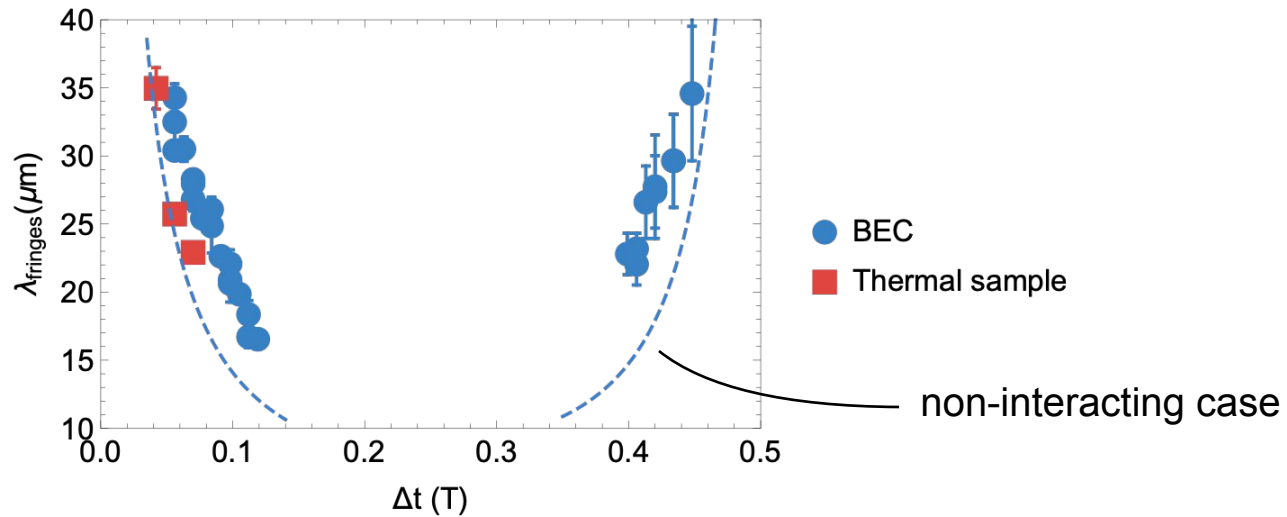
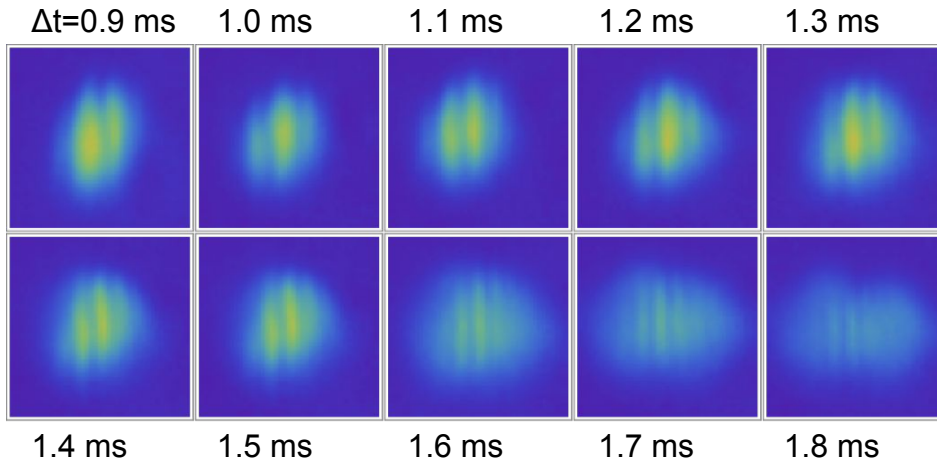
GPE simulation (with M. Modugno, Bilbao)



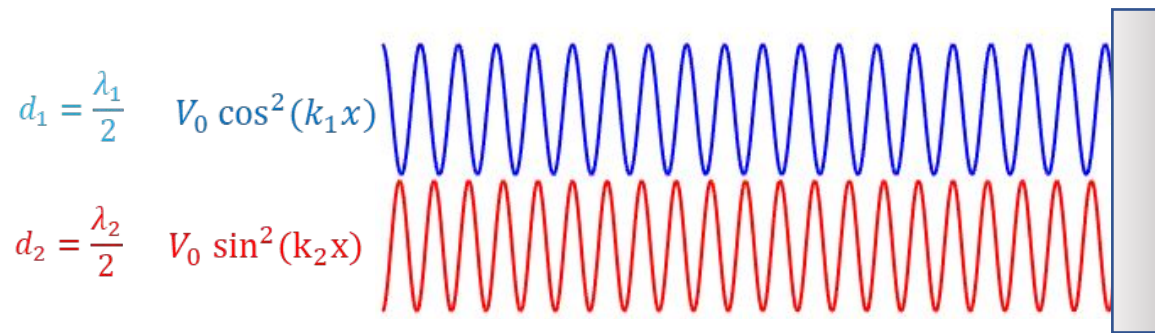
$$k_{\text{fringes}} = \frac{m}{\hbar} \frac{1}{1 + \omega^2 t_{\text{TOF}}^2} \left( \delta v + \frac{\delta x}{t_{\text{TOF}}} \omega t_{\text{TOF}} \right)$$

- experiment
- simulation
- - - simulation non-interacting





Small wavevector lattice developed by L. Masi, T. Petrucciani and M. Fattori,  
benefit: reduce  $v_R$  hence oscillation amplitude, hence anharmonicity

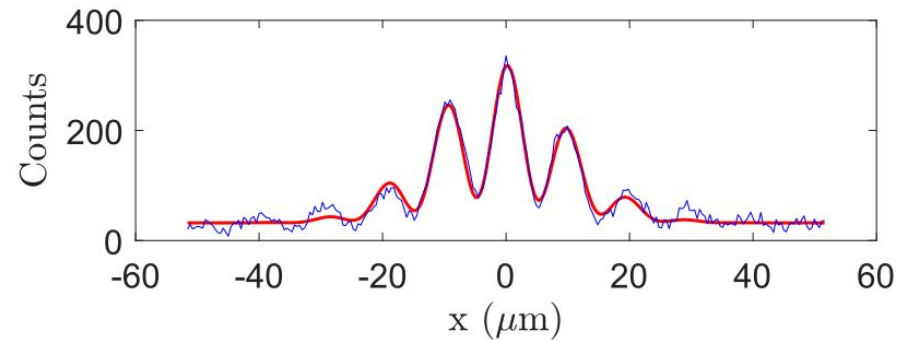
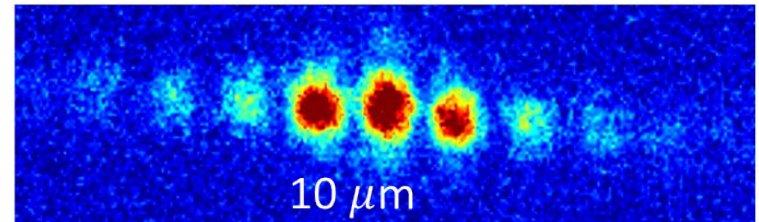
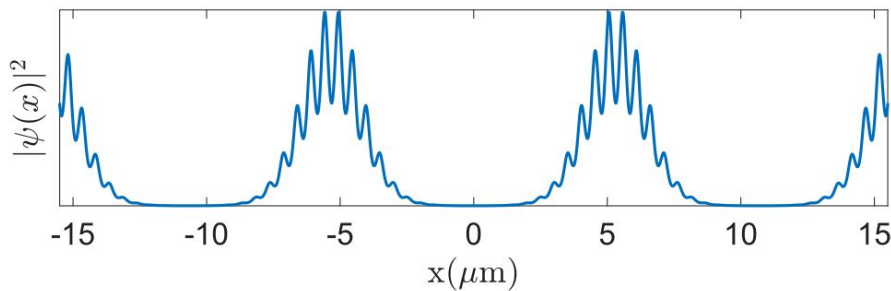
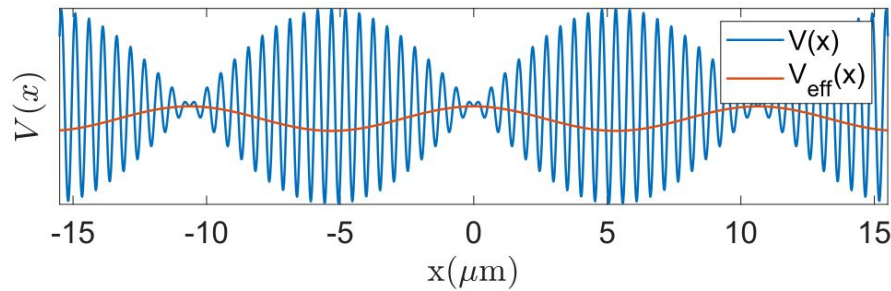


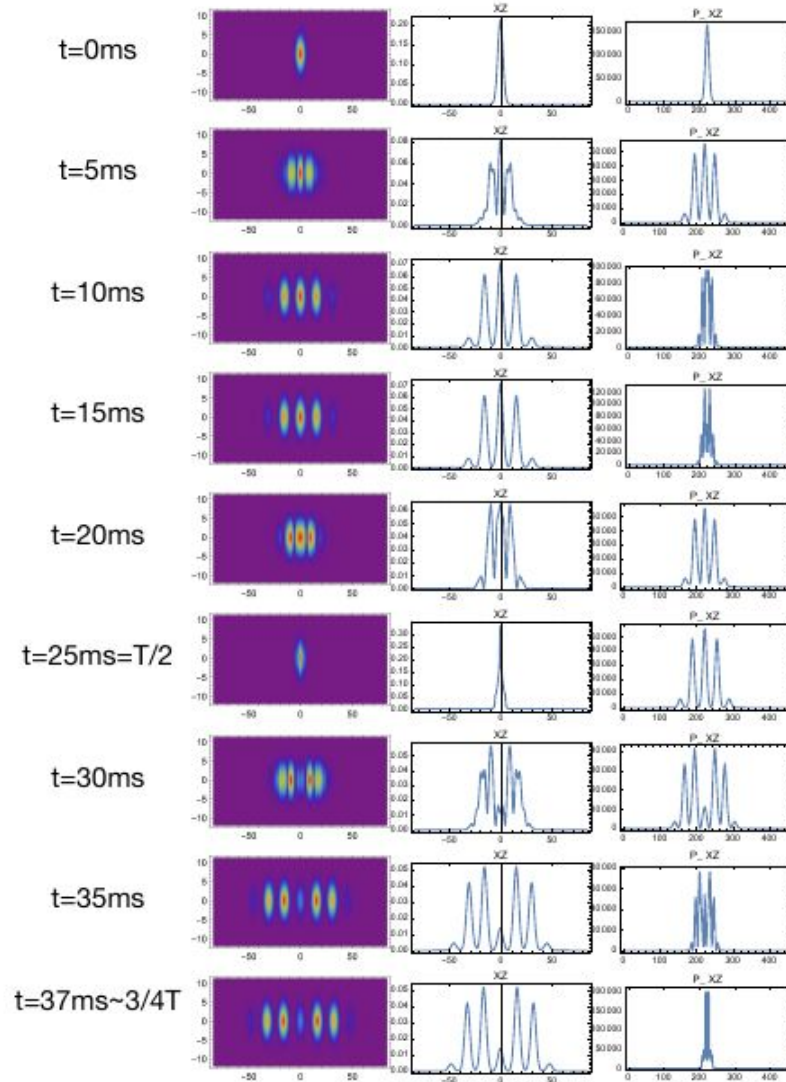
Beat-note between two commensurate wavelengths,  $n$

$$n \lambda_1 = (n + 1) \lambda_2$$

$$d = \frac{n \lambda_1}{2}$$

Potential equivalent to a large spacing optical lattice





$^{39}\text{K}$  benefit:

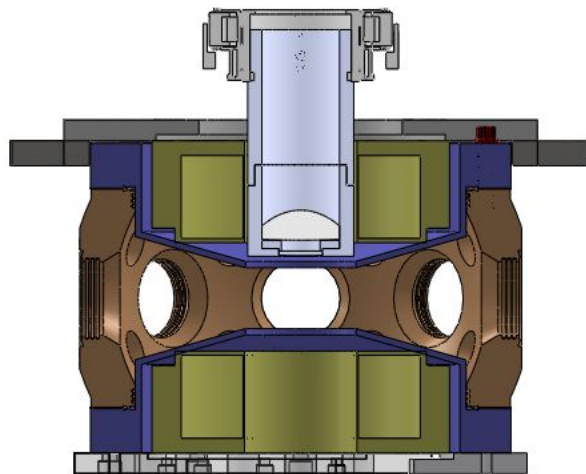
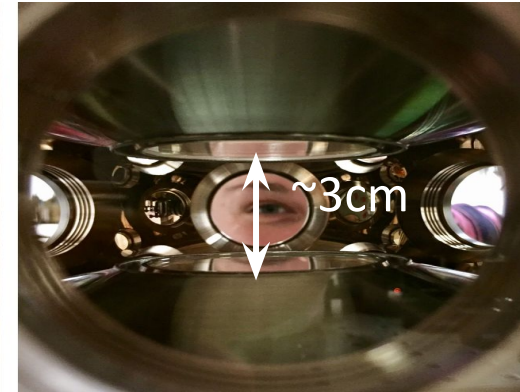
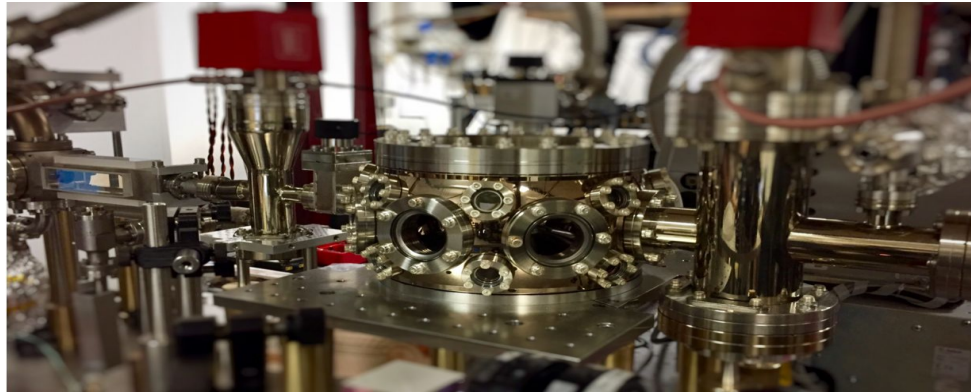
control of interactions

Currently performing numerical simulations of Kapitza-Dirac trapped interferometer

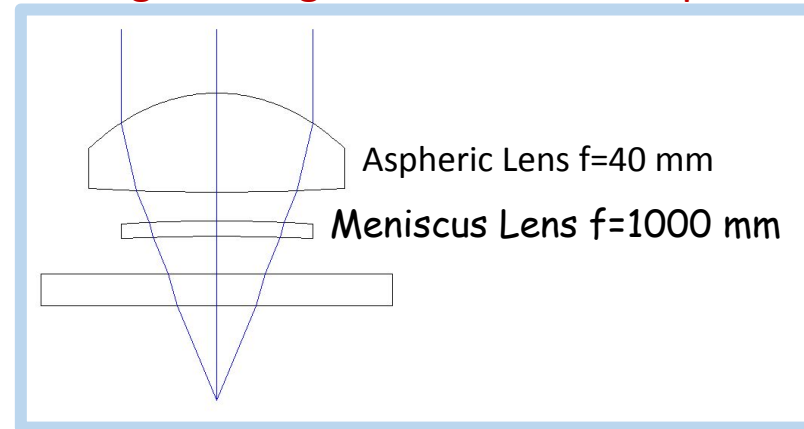
### T2.4: Arbitrarily filtered, multimode, Kapitza-Dirac interferometer (CNR; M1-M18)

The multiple momentum components corresponding to the comb photonic modes will be obtained from a single trapped Bose-Einstein condensate (BEC), by means of a Kapitza-Dirac pulse.

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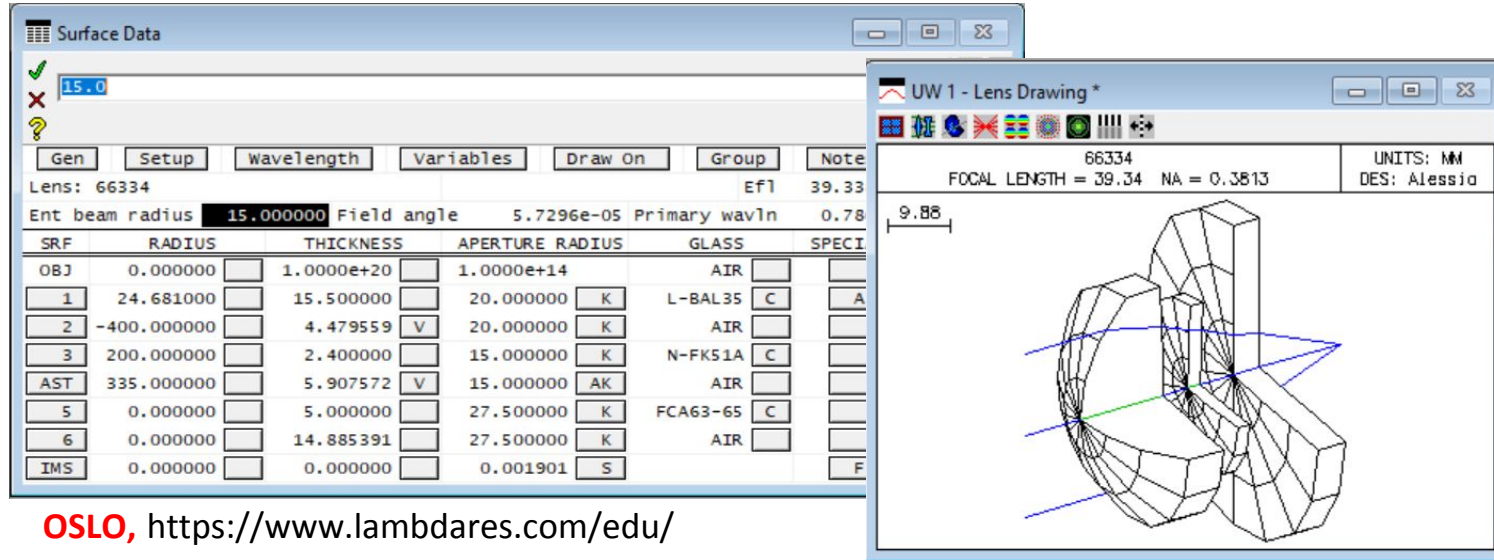
## Long working distance microscope



Diffraction-limited resolution:

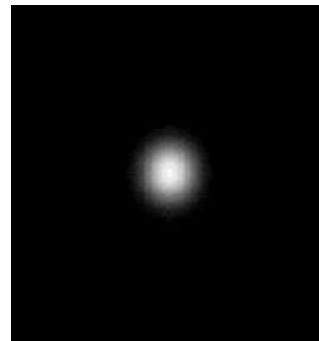
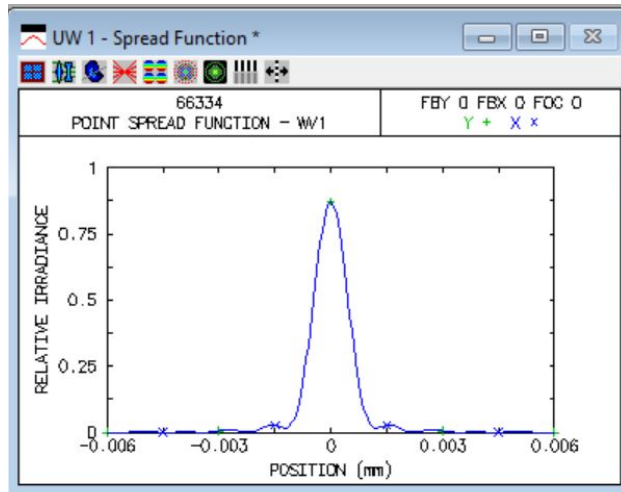
$$r = \frac{1.22\lambda}{2n \sin \theta} = \frac{0.61\lambda}{NA} = 1.2\mu\text{m}$$

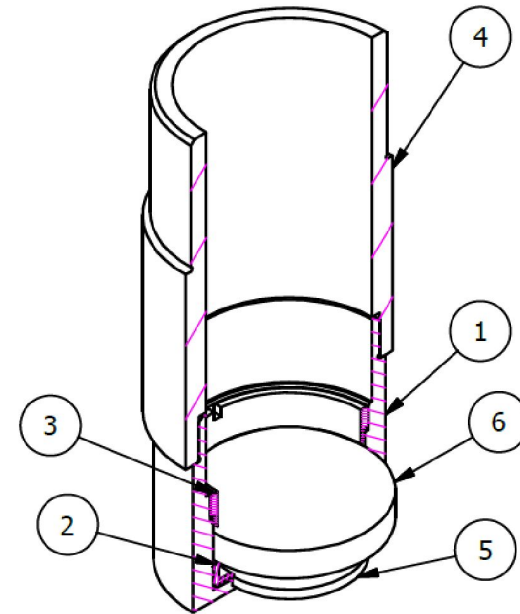
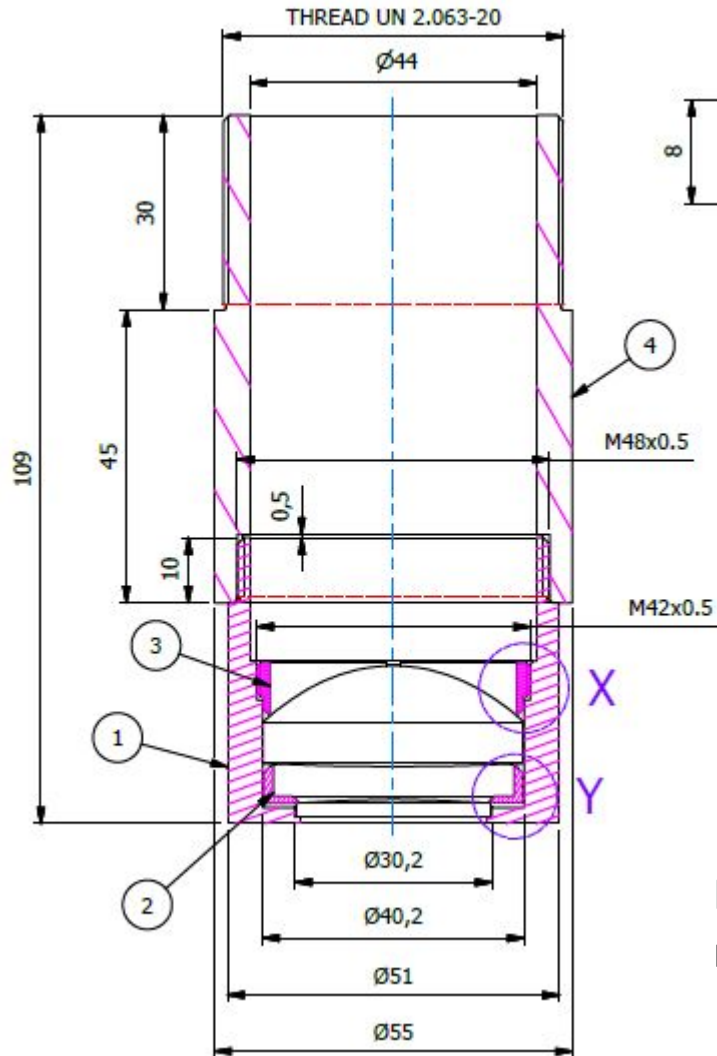




OSLO, <https://www.lambdare.com/edu/>

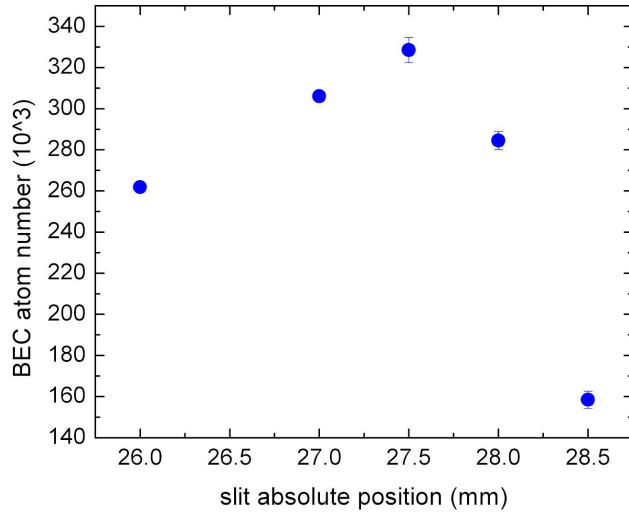
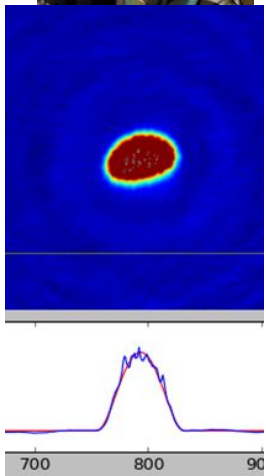
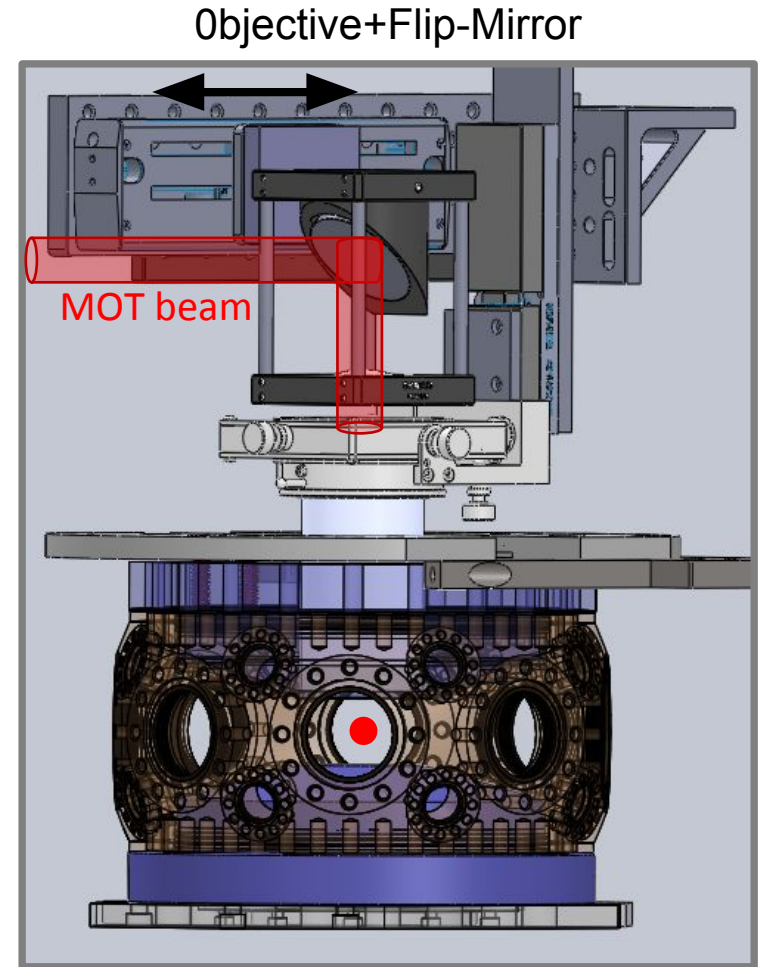
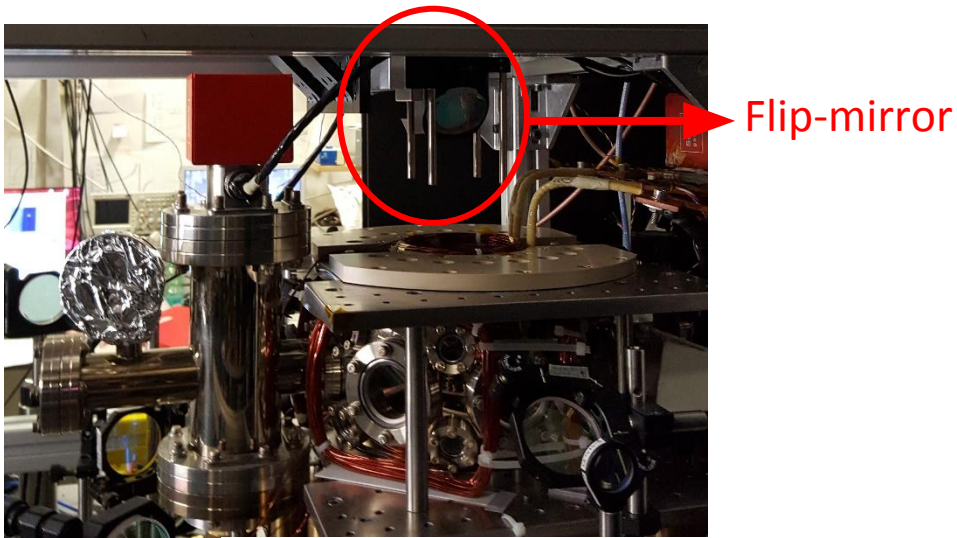
## Point Spread Function





Presence of time-varying magnetic fields:  
rigid plastic material, PEEK





- **D2.3: Implementation of Kapitza-Dirac multi-mode beam-splitter and interferometer (CNR, M18)**

Kapitza-Dirac multimode: anharmonicity detected and explained

Bragg two modes in trap demonstrated

Fringe spacing explained with numerical simulations, phase stability under investigation

Novel large spacing lattice and  $^{39}\text{K}$  to control interactions

- DMD momentum filter

Design and assembly of objective, under test now

Modified setup to integrate objective

**Following are back-up slides**

## Evolution in Trap for short times

At  $t = 0$  *first pulse*

at  $\Delta t < \frac{1}{\omega}$  *second pulse* ( $\Delta t \approx 1 \text{ ms}$ ,  $\omega = 2\pi \cdot 70 \text{ Hz}$ )

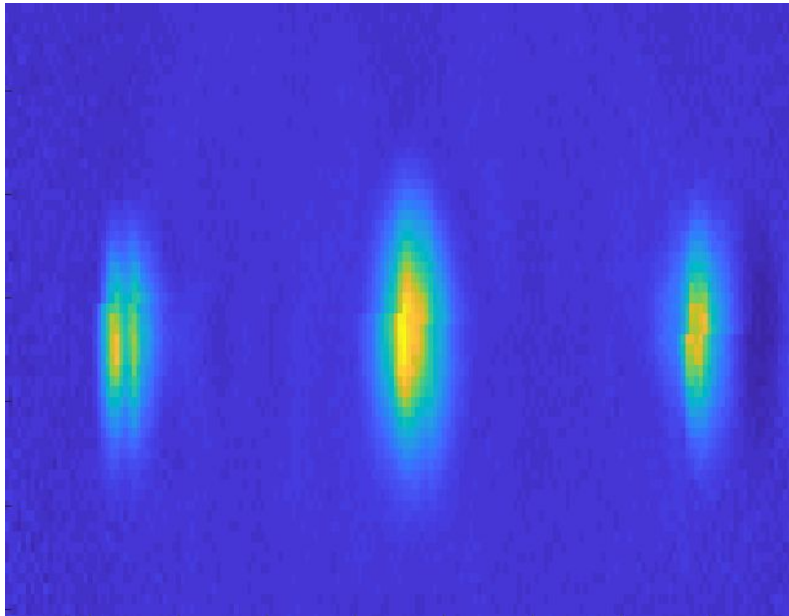
$$x(t) = \frac{2v_r}{\omega} \sin(\omega t) \approx 2v_r t,$$

$$v(t) = 2v_r \cos(\omega t) \approx 2v_r$$

$$k = \frac{m}{\hbar} \frac{1}{1 + \omega^2 t_{\text{TOF}}^2} \left( \delta v + \frac{\delta x}{t_{\text{TOF}}} \omega t_{\text{TOF}} \right)$$

$$\delta v = 0 \text{ e } t_{\text{TOF}} > \omega^{-1} \rightarrow k = \frac{md}{\hbar t_{\text{TOF}}}$$

Interference fringes after 32 ms TOF



$$d = 2v_r \Delta t = 7.6 \mu\text{m}$$

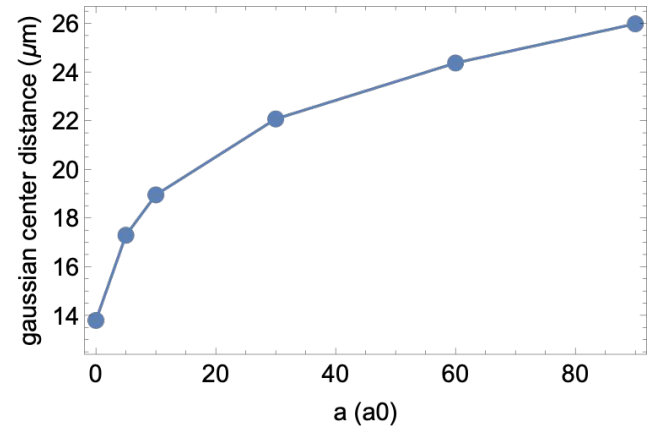
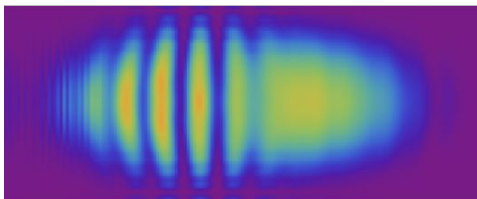
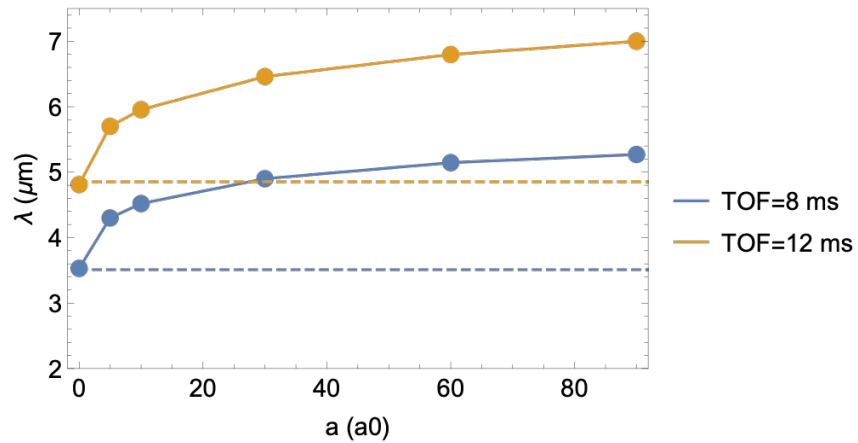
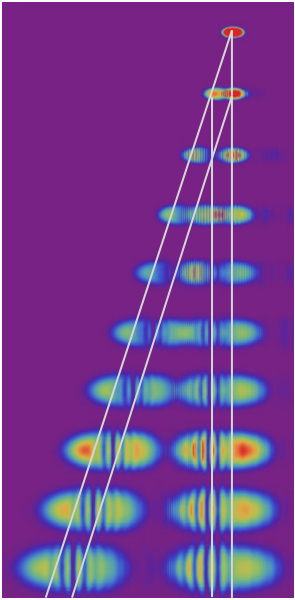
$$\lambda = \frac{\hbar t}{md} = 19.2 \mu\text{m}$$

# Numerical simulations for different interactions

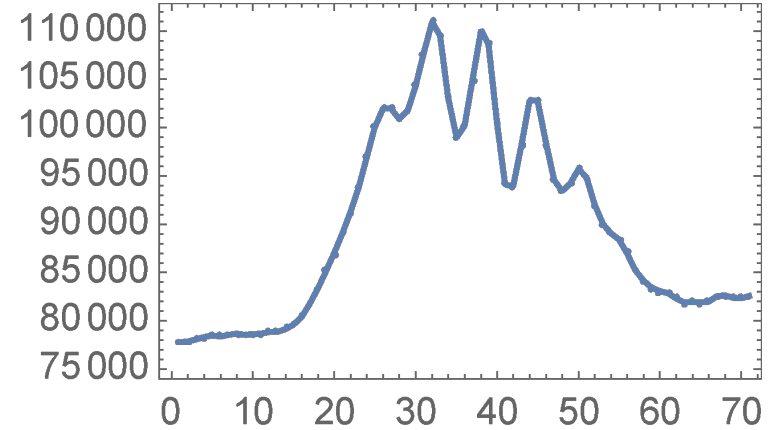
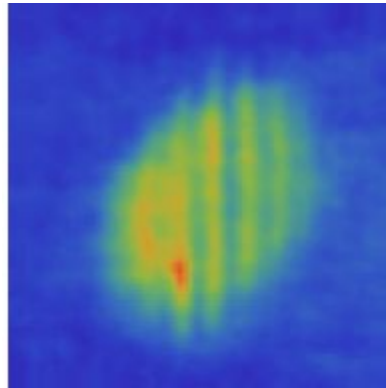
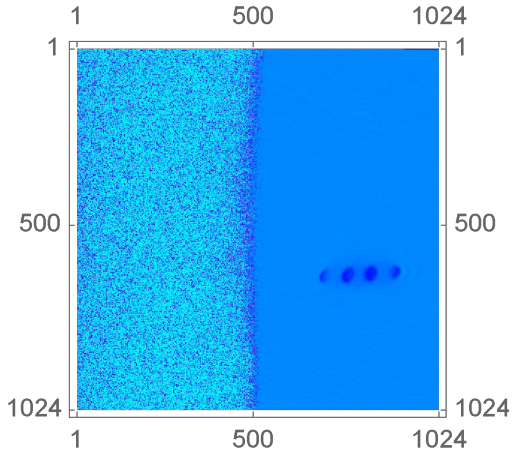
change scattering length  $a$ , fix delay and  $\Delta t$

effects of interactions:

- fringes spacing
- fringes shape
- condensates positions



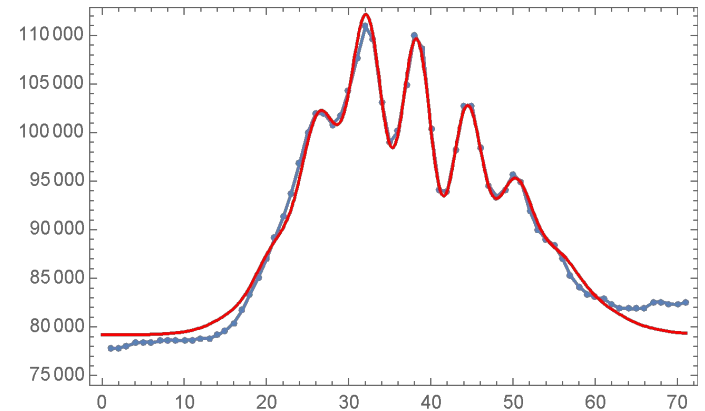
# Image fit, interferometer phase



$$n_1 = \frac{A_1^2}{\sqrt{2}\sigma_1} e^{-\frac{(x-x_0)^2}{2\sigma_1^2}}$$

$$n_2 = \frac{A_2^2}{\sqrt{2}\sigma_2} e^{-\frac{(x-(x_0+d))^2}{2\sigma_2^2}}$$

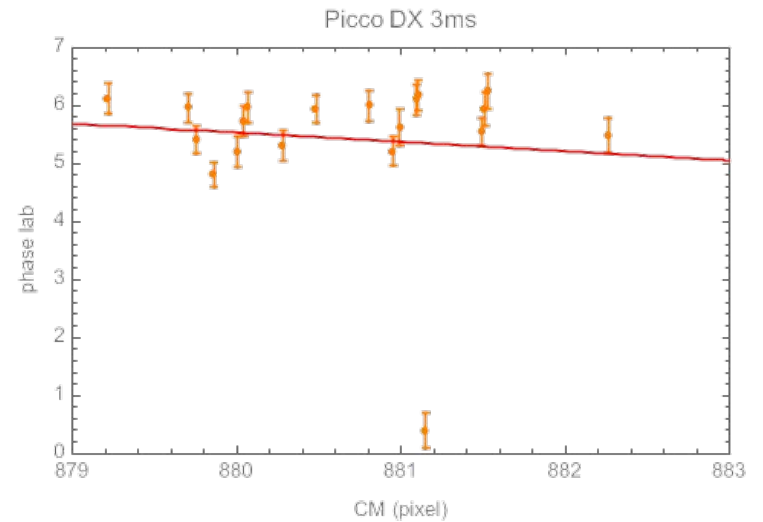
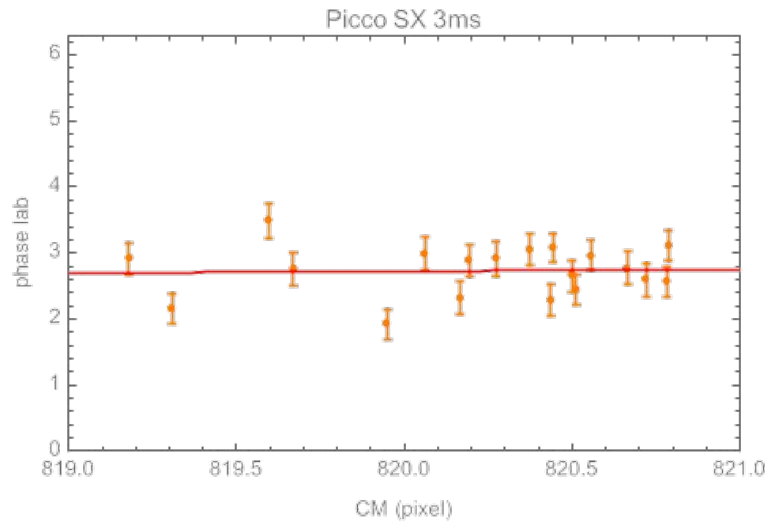
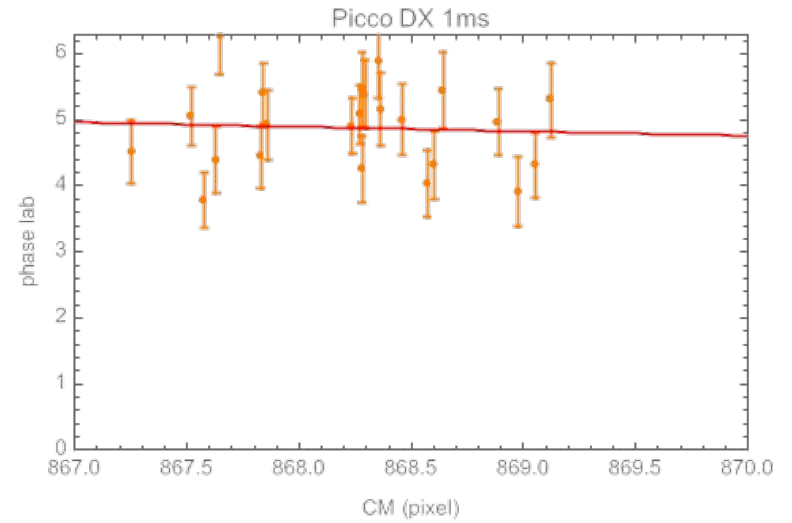
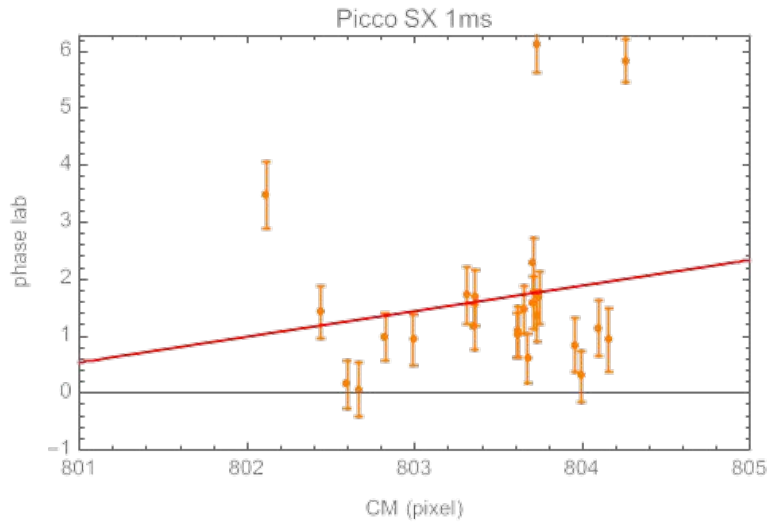
$$f_{FIT} = n_1 + n_2 + 2|C|\sqrt{n_1 n_2} \cos\left(\frac{2\pi}{\lambda}x + \phi\right)$$



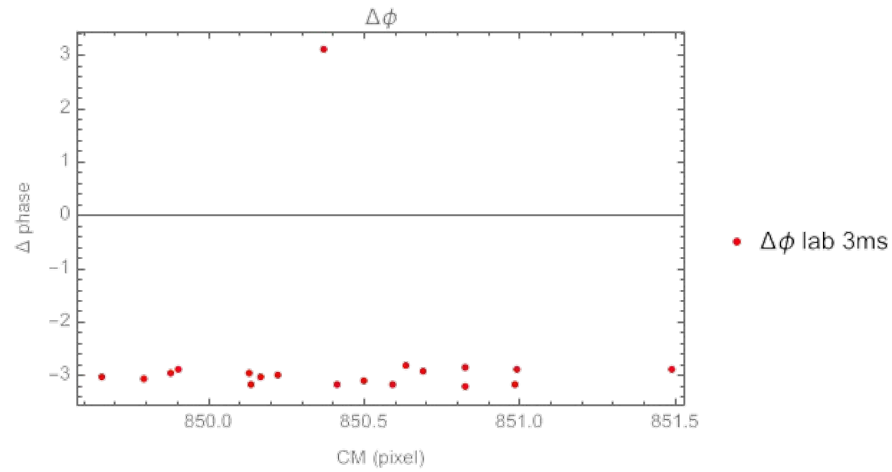
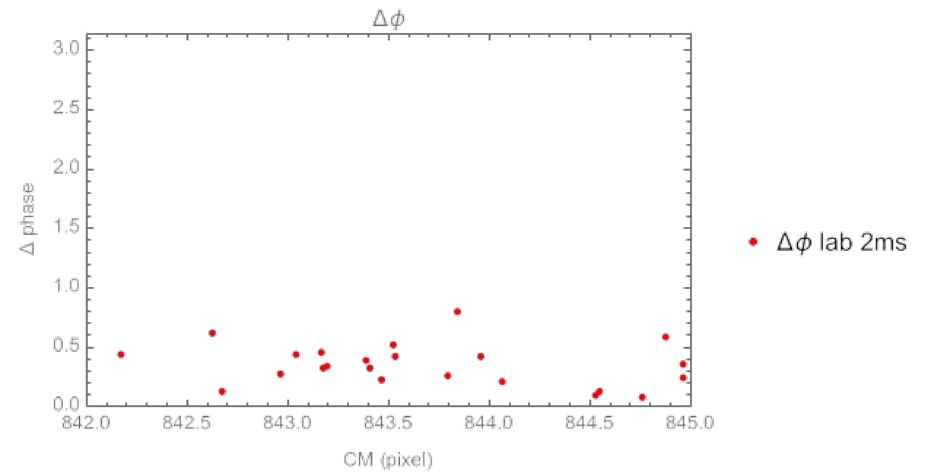
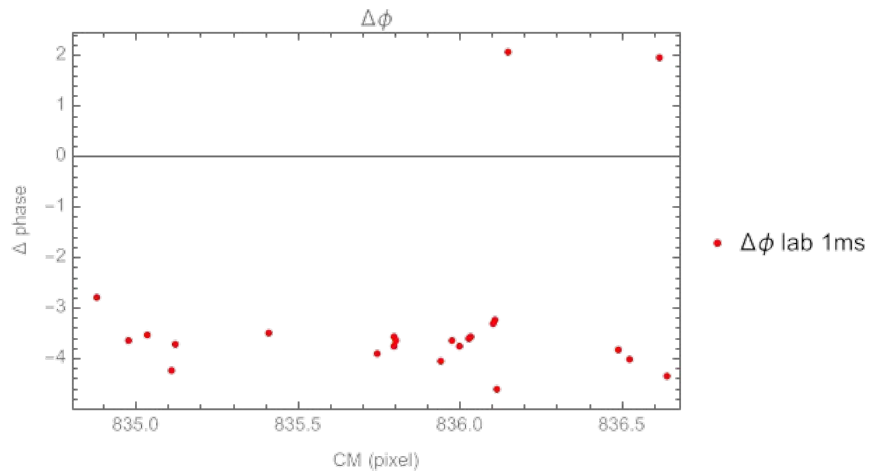
$A_1, \sigma_1, A_2, \sigma_2, x_0, d, C, \lambda, \phi$



# Interferometer phase, Bragg ToF



# Port comparison, Bragg ToF





# Phase – Bragg in trap

